it he gives the development of those computational techniques that he devised for the relocation of the planetoid Ceres. Because of the few existing observations, and the unusual elements of its orbit, existing methods did not suffice. But from Gauss's calculations, as he proudly announced, Ceres was rediscovered "the first clear night".

The appendix contains an account of other computational techniques by Encke and Peirce, and 39 pages of tables by Le Verrier, Bessel, and others to facilitate certain astronomical calculations.

D. S.

85[R].—I. Todhunter, *A History of the Mathematical Theories of Attraction and the Figure of the Earth*, Dover Publications Inc., New York, 1962, xxxvi + 984 p., 22 cm. Price $7.50.

This is a timely and most welcome reprint of Todhunter's history, which was originally published in 1873. In it he gives a detailed and critical account of all the work in this field from the time of Newton to that of Laplace. This includes that of Newton, Huygens, Maupertius, Clairaut, Maclaurin, D'Alembert, Boscovich, Laplace, Legendre, Poisson, Ivory, and others.

The volume is not only of current physical interest but also contains valuable historical accounts of the origins of potential theory and of many investigations in partial differential equations. The style is simple and pleasant, and is enlivened by classical descriptions and original observations of his own. Thus: "Maupertius . . . who flattened the poles and the Cassinis"; "Madame la Marquise du Chastellet . . . from the fluctuation of her opinions, it seems as if she had not yet entirely exchanged the caprice of fashion for the austerity of science"; and "Gauss's writings are distinguished for the combination of mathematical ability with power of expression: in his hands Latin and German rival French itself for clearness and precision."

For the hurried reader the long preface and table of contents give a good idea of the volume's scope.

D. S.

86[S, X].—S. L. Sobolev, *Applications of Functional Analysis in Mathematical Physics*, Volume Seven, *Translations of Mathematical Monographs*, American Mathematical Society, Providence, Rhode Island, 1963, viii + 239 p., 24 cm. Price $6.70.

The development of the theory of distributions and generalized functions has its roots in the works of many famous mathematicians, such as J. Hadamard, M. Riesz, S. Bochner, and J. Leray, to mention a few. In 1936, S. L. Sobolev introduced a concept of generalized functions and derivatives which is essentially equivalent to the one now used. However, it was only with the appearance of L. Schwartz's comprehensive books on distributions in 1950 and 1951 that the field began to receive the systematic and extensive treatment we now know.

In the same year, namely 1950, S. L. Sobolev published the original Russian edition of the present monograph. It was the outgrowth of courses given at Leningrad State University and presented a unifying treatment of a number of problems in partial differential equations, using Sobolev's own approach to the concepts of generalized functions and their applications. The book appears not to have been noticed much by the non-Russian speaking mathematical world. In turn, the presentation in the book does not take the least bit of notice of any related work in the Western world.

In the 13 years that have elapsed between the publication of the original and this translation, a large body of knowledge has developed concerning the application of the theory of generalized functions to differential equations. Sobolev's earlier approaches to these problems and his results now appear in one way or another as part of a systematic and comprehensive theory. Nevertheless, this monograph is a veritable classic of mathematics; it is the work of a great mathematician who presents his own unified treatment of an important class of problems, with great clarity and without any attempt at being as general as possible. As such, it certainly remains an eminently worthwhile work to read.

Chapter I provides the necessary functional-analytic background and begins with a brief discussion of L_p spaces. Next, generalized derivatives are introduced; the definition is essentially identical to that for distributions, except that the generalized derivative of a function is itself a summable function. The linear space of all summable functions having on a finite domain Ω all generalized derivatives of order l summable to power $p > 1$ is denoted by $W_p^{(l)}$. $W_p^{(l)}$ with the norm

$$\| u \|_{W_p^{(l)}}^p = \| u \|_{L_p}^p + \int_\Omega \left[\sum_{|q|=l} | D^q u |^2 \right]^{p/2} d\omega$$

are the spaces upon which the discussions of all following parts of the book are based. The remainder of Chapter I is devoted to an investigation of the properties of $W_p^{(l)}$. Two theorems play an essential role here, assuring that under certain conditions $W_p^{(l)}$ can be imbedded in C or in some L_{q^*}, and that in both cases the imbedding operators are completely continuous. These theorems also give a natural setting for some inequalities between different norms on $W_p^{(l)}$.

Chapter II, entitled Variational Methods in Mathematical Physics, begins with an application of the theory of Chapter I using variational methods to prove the existence and uniqueness of the solution of Dirichlet's problem for the Laplace equation in the space $W_2^{(1)}$. This is then continued with a similar approach to the existence and uniqueness of the solutions of the Neumann problem and of the basic boundary value problem for the polyharmonic equation. The chapter ends with a detailed discussion of the eigenvalue problem

$$\Delta u + \lambda u = 0, \qquad \left. \frac{\partial u}{\partial n} \right|_S - hu |_S = 0$$

Chapter III concerns the solution of the Cauchy problem in $W_p^{(l)}$ spaces for the n-dimensional wave equation and for linear hyperbolic equations with variable coefficients. This discussion is based on the classical integration theory for these equations under the assumption of sufficiently smooth data. All necessary results are fully developed using the theory of characteristics for the $(2k + 1)$-dimensional case and Hadamard's method of descent for the space of $(2k)$-dimensions. The results for the generalized Cauchy problem then follow naturally from the classical results and the properties of the $W_p^{(l)}$ functions.

There is no question that the book plays an even more unique and distinct role today than at the time of its first appearance. It should certainly not be compared with any of the newer systematic and comprehensive treatises of the field, such as L. Hörmander's excellent work (Springer-Academic Press, 1963). As stated before, the value of this monograph is rather the clear and detailed presentation of one unified and original approach to the solution of some of the basic problems in the theory of partial differential equations, even though this approach has now become part of a larger theory. It appears to this reviewer that the value of the translation might have been enhanced even more if an up-to-date, annotated bibliography had been provided to supply the student with the necessary bridge to the present state of the field.

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87[X].—L. Fox, *Numerical Solution of Ordinary and Partial Differential Equations*, Addison-Wesley Publishing Company, Inc., Reading, Mass., 1962, ix + 509 p., 23.5 cm. Price $10.00.

This volume is another useful addition to the expanding library in the field of numerical analysis devoted to the solution of differential equations by numerical methods. The material is based on a series of lectures presented at the Oxford University Computing Laboratory during the summer of 1961. The areas covered include the following: (1) ordinary differential equations; (2) integral equations; (3) introduction to partial differential equations; and (4) practical problems in partial differential equations.

Although the lectures were delivered by a number of workers in the field, the book achieves a remarkable degree of coherence. The editor, L. Fox, and the contributors (D. F. Moyers, *et al.*) also deserve much credit for the promptness of the publication and the lucidity of the presentation.

The first section treats the solution of ordinary differential equations by the method of finite differences. It covers such topics as the Runge-Kutta method, eigenvalue problems, and Chebyshev approximation. Of special interest is the discussion of stability as it relates to the solution of ordinary differential equations. The author finds it useful to classify several types of instability, such as inherent instability, partial instability, and strong instability.

Section 2 discusses the numerical solution of integral equations, including such topics as Fredholm equations of the first, second, and third kinds, equations of Volterra type, integro-differential equations in nuclear collision problems, and the Hartree-Fack equation.

Section 3 contains a readable exposition of the methods in common use for the numerical solution of partial differential equations of hyperbolic, elliptic, and parabolic types.

Section 4 contains a discussion of illustrative problems involving partial differential equations solved by the methods of finite differences, selected from a representative cross-section of modern physics and engineering.

Although generally well done, the book does show the signs of haste in many spots, and should be improved in later editions.

H. P.